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UNEMPLOYMENT AND PRICE DYNAMICS IN A  
MONETARY-FISCAL POLICY MODEL

by

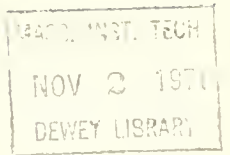
Duncan K. Foley

Number 80 - ~~September~~ <sup>October</sup> 1971

**massachusetts  
institute of  
technology**

**50 memorial drive  
cambridge, mass. 02139**





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UNEMPLOYMENT AND PRICE DYNAMICS  
IN A MONETARY-FISCAL POLICY MODEL

In a recent paper [1] and book [2] Miguel Sidrauski and I developed a full employment continuous-time model of an economy controlled indirectly by monetary and fiscal policy. The close relation of this model to the General Theory of Keynes [4] and to some traditional versions of macroeconomic theory was somewhat obscured by the fact that we analyzed only a full employment version of the model. In this paper it is my purpose to extend that model to include unemployment. The strict treatment of stocks and flows in continuous time characteristic of the original monetary-fiscal policy model has consequences for unemployment as well; it unravels some logical confusions about the role of price and wage flexibility in unemployment models and illuminates the theoretical function of a theory of price dynamics.

In section 1, I outline the basic monetary-fiscal policy model; this review assumes some familiarity with Part I of our earlier book. In section 2, I discuss the problem of static or instantaneous unemployment equilibrium, the distinction between disequilibrium of relative prices and disequilibrium of money prices, the consequences of unemployment in assets markets and what might be called the static accelerator. In section 3, I discuss the dynamic version of the model and describe the theoretical role of a theory of price dynamics. In section 4, I begin to discuss the formation of a theory of price dynamics based

on the notions of rational expectations and stochastic process theory.

### Section 1: The Basic Model

The monetary-fiscal policy model follows a central macroeconomic tradition (most familiar in the ideas of the IS-LM analysis [3]) in analyzing macroeconomic equilibrium as the interaction of two kinds of markets: assets markets for existing stocks of money, bonds, and capital; and a consumption market for the flow of output from the consumption sector.

The production model is the standard two-factor two-sector production model, with constant-returns to scale production functions in investment and consumption sectors. The first degree homogeneity permits me to write all market-clearing conditions in terms of intensive per capita quantities. At each instant, rentals to capital and wages must be the same in each sector given the relative price,  $p_k$ , of investment goods in consumption goods units.<sup>1</sup>

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<sup>1</sup>Throughout I use consumption goods as numéraire. Just as  $p_k$  is the price of investment goods in terms of consumption goods so  $p_m$  is the price of money in terms of consumption goods, the inverse of the price level. The wage  $w$  and rental  $r$  are both measured in consumption good units, so that  $r/p_k$  is a pure number, the own rate of return to capital.

---

$$(1.1) \quad r = f'_c(k_c) = p_k f'_I(k_I)$$

$$(1.2) \quad w = f_c(k_c) - k_c f'_c(k_c) = p_k (f_I(k_I) - k_I f'_I(k_I))$$

where  $f_c, f_I$  are the intensive production functions in the consumption and investment sectors respectively, and  $k_c$  and  $k_I$  are the capital intensities in each sector.<sup>1</sup> These equations suffice to determine  $k_c$  and  $k_I$ , but the actual outputs depend on the distribution of labor between the two sectors, which is determined by market clearing in the labor and capital markets:

$$(1.3) \quad l_c k_c + l_I k_I = k$$

$$(1.4) \quad l_c + l_I = 1,$$

---

<sup>1</sup>We always assume  $k_c > k_I$ .

---

where  $l_c, l_I$  are the proportions of the labor force employed in each sector, and  $k$  is the total capital stock divided by the total labor force. Since these equations imply full employment they will have to be revised when I come to introduce unemployment.

The solution of these equations can be summed up as supply functions  $q_c(k, p_k), q_I(k, p_k)$  showing the rate of output in each sector for given  $k$  and  $p_k$ , and a function  $r(p_k)/p_k$  showing the own rate of return to capital for given  $p_k$ .

In the assets market the real supplies of the three assets are fixed at any instant, and the demands depend on desired portfolio balance among them, which in turn involve non-human total net worth,  $a = p_m g + p_k k$ , where  $g$  is the nominal value of the government debt, output  $q(k, p_k) = q_c(k, p_k) + p_k q_I(k, p_k)$  which represents the transactions demand for money, and expected real rates of return to the assets. For money the

real rate is  $\pi_m$ , the expected rate of deflation, for bonds  $i + \pi_m$ , where  $i$  is the interest rate, since we assume bonds to be instantly redeemable at a fixed money price, like a savings account, and for capital  $r(p_k)/p_k + \pi_k$ , where  $\pi_k$  is the expected rate of change in  $p_k$ .

We write the asset market clearing in three equations:

$$(1.5) \quad p_m m = L(a, q(k, p_k), \pi_m, i + \pi_m, r(p_k)/p_k + \pi_k)$$

$$(1.6) \quad p_m h = H(a, q(k, p_k), \pi_m, i + \pi_m, r(p_k)/p_k + \pi_k)$$

$$(1.7) \quad p_k k = J(a, q(k, p_k), \pi_m, i + \pi_m, r(p_k)/p_k + \pi_k)$$

where  $h$  is the nominal supply of government bonds and  $H$  the net demand of the private sector to hold bonds. These demands must satisfy a wealth constraint:

$$(1.8) \quad L + H + J = a,$$

since at any instant in revising portfolios people can only buy one asset by selling another.

For a given  $p_m$ , equations (1.5) - (1.7) can be solved to find market clearing  $p_k$  and  $i$ . The relation between  $p_m$  and  $p_k$  that clears the assets markets we call the  $aa$  schedule: under mild assumptions it is upward sloping (see [2] Ch. 3). The  $aa$  schedule is a more general analogue of the  $LM$  schedule.

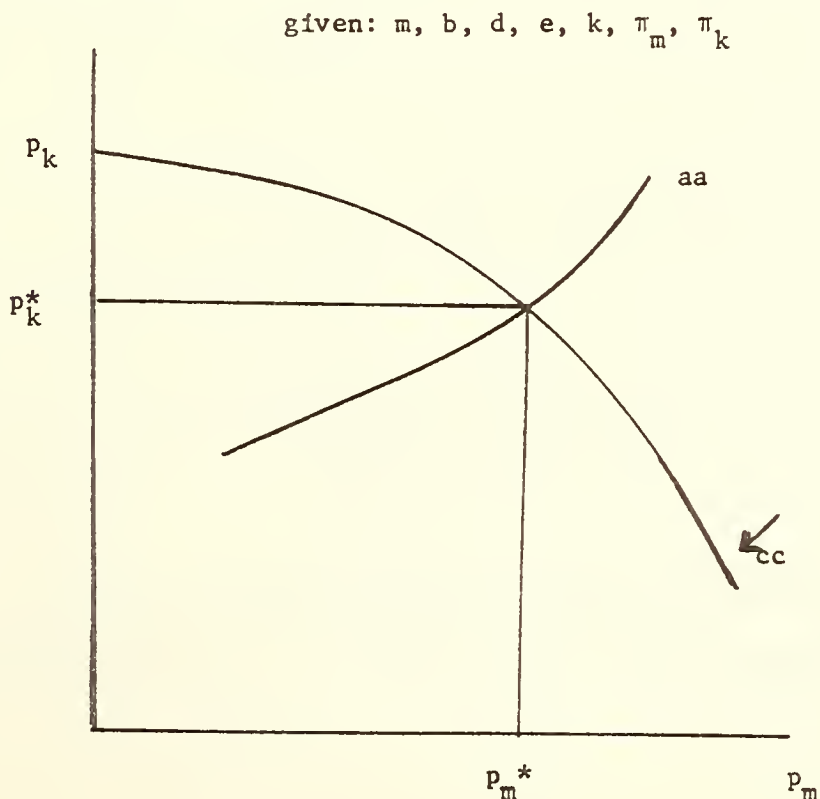
The instantaneous market clearing model is closed by the addition of a consumption function relating disposable income, wealth, and government expenditures to total demand for the flow of consumption goods. The market clearing condition in this flow market is:

$$(1.9) \quad q_c(k, p_k) = c^d(a, q(k, p_k) + p_m d - e + \pi_m p_m g + \pi_k p_k k) + e.$$

The reader can quickly verify that if  $e$  is government expenditures (assumed to be all consumption goods) and  $d$  is the nominal deficit then  $p_m d - e$  is net real transfers less taxes. The terms  $\pi_m p_m g$  and  $\pi_k p_k k$  represent anticipated capital gains, which we include in disposable income.

Again, for given  $p_m$ , (1.9) can be solved for the  $p_k$  that just induces a flow supply of consumption goods equal to the flow demand. This relation between  $p_m$  and  $p_k$  we call the  $cc$  schedule: it is frequently downward-sloping. The analogy is to the IS curve.

The total instantaneous equilibrium of the economy can be pictured as the intersection of the  $aa$  and  $cc$  curves (see Figure 1).



The situation pictured in Figure 1 can be described in words as follows: at time  $t$ , given that the government is running a deficit  $d$  with expenditures  $e$ , that the monetary authorities have divided the government debt in a supply  $m$  of high-powered money and  $b$  of bonds, that the accumulated capital stock is  $k$  and that expectations of changes in the relative prices of money and capital are  $\pi_m$  and  $\pi_k$ , if the price of money were  $p_m^*$  and the price of capital  $p_k^*$ , asset holders would be content with their portfolios and the supply and demand for the flow of consumption goods would be equal.

Government policy, by changing  $d$  and  $e$ , can shift the  $cc$ , or by open market operations can alter  $m$  and  $b$  and shift the  $aa$ . Over time, investment will alter  $k$ , the deficit will change the sum  $(m + b)$  and experience will modify  $\pi_m$  and  $\pi_k$ ; as a result both  $cc$  and  $aa$  will move. It is not difficult to set up a system of differential equations to represent the dynamics of this system. For convenience I will summarize them (where  $n$  is the labor force growth rate):

(1.10) $p_m m = L(a, q(k, p_k), \pi_m, i + \pi_m, r(p_k)(p_k + \pi_k))$	] Instantaneous Equilibrium Conditions
(1.11) $p_k k = J(a, q(k, p_k), \pi_m, i + \pi_m, r(p_k)(p_k + \pi_k))$	
(1.12) $q_c(k, p_k) = c^d(a, y^d) + e$	
(1.13) $\dot{k} = q_I(k, p_k) - n k$	] Laws of Accumulation
(1.14) $\dot{g} = d - n g$	

To these five must be added two equations determining the formation of expectations, that is, of  $\pi_m$  and  $\pi_k$ , and three representing either paths

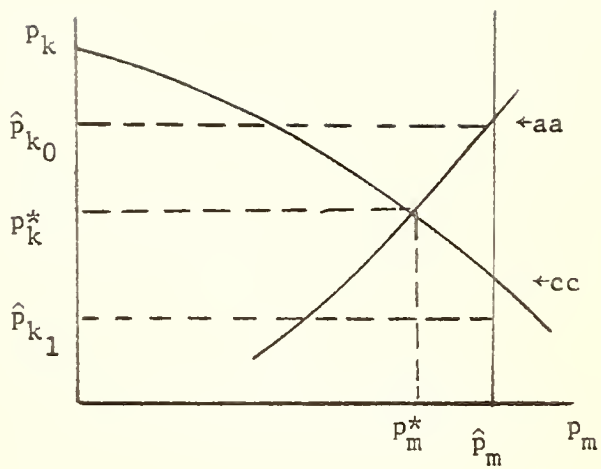
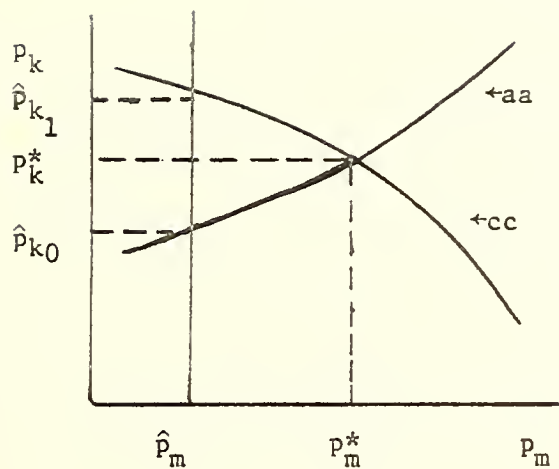
or goals for government policies (since there are three government tools, the level of expenditures, the size of the deficit, and the composition of the debt between money and bonds.) For a detailed analysis of several such systems, see [2].

## Section 2: Unemployment

Let me call the value of  $p_m$  that satisfies (1.10) - (1.12) the full-employment price of money. By assuming that the actual price of money,  $\hat{p}_m$ , is at every instant equal to the full-employment price of money,  $p_m^*$ , we rule out any difficulties of maintaining full employment and any discrepancy between full-employment plans for savings and investment. The actual price of money will at every instant reconcile these plans.

Suppose now that for some reason  $\hat{p}_m$  does not instantaneously equal  $p_m^*$ . Whether we should expect it to or not I will discuss in succeeding sections. We have two possibilities, a  $\hat{p}_m$  below  $p_m^*$ , as in Figure 2-1 or a  $\hat{p}_m$  above  $p_m^*$  as in Figure 2-2.

Given:  $m, b, k, d, e, \pi_m, \pi_k$



In each of these cases we have the difficulty to which Keynes pointed when he insisted that saving and investment plans, being made by quite different people, will not necessarily coincide. In Figure 2-1, for example, wealth holders are content to own the existing stock of capital only at price  $\hat{p}_{k0}$ . On the other hand savers want at full employment to save so much that a higher price,  $\hat{p}_{k1}$  would be necessary to induce enough investment to equal their desired savings. To put it another way, at  $\hat{p}_{k0}$ , the going price of capital in the assets markets, the flow supply of consumption goods exceeds what people want to buy. This is the situation of glut, recession, or depression.

If the actual price of money will not adjust instantaneously to  $p_m^*$ , but stays at  $\hat{p}_m$ , what price of capital will rule? At this point many possibilities open up, but I choose what seems to me to be the simplest and most plausible stylized assumption. I assume that the assets markets remain in equilibrium (or at any rate close to it) so that the economy will stay on the aa curve. This is simple because it confines the disequilibrium



to one market, the flow market for consumption goods. It is plausible to the extent that asset markets are better organized, quicker to respond and suffer fewer restrictions than markets for labor or consumption goods.

Notice that I really have only one market out of equilibrium, not two. In particular an excess flow supply of consumption goods does not imply an excess demand for stocks of money: the wealth constraint and income constraint do not interact. At this point many people may ask, "But what about the flow demand for money?" As we try to make clear in our book, the concept of a flow demand for money or capital is not easy to define, since it depends on expected future prices; and in general when it is defined, the flow demands and supplies for assets will not be equal even if the other markets (for stocks of assets and flows of consumption goods) are cleared. In the present case it will be true that total planned full-employment saving will exceed the sum of planned investment and government deficit. This is the familiar discrepancy between planned saving and investment.

What happens in the consumption market when a situation like Figure 2-1 or Figure 2-2 arises? In the real world there appear to be three responses to a situation like Figure 2-1 where planned saving at full employment exceeds planned investment and an excess flow supply of consumption goods threatens. First, for short periods, inventories will be accumulated, but this policy is only good against episodes of very short duration. Second, firms will reduce productivity, utilising labor and capital less intensively. Third, firms will begin to disemploy labor and capital. This final step is the only one I will explicitly model here, but it is important to keep the

others in mind.

If firms reduce employment they reduce the output of consumption goods for any given  $p_k$ , and if they reduce it enough they can clear the market for consumption goods at the assets-market equilibrium price of capital  $\hat{p}_{k0}$ .

In the opposite situation of Figure 2-2, the options are symmetrical. For a short episode inventories can be run down to meet the excess demand. For a longer one firms may be able to achieve super-normal productivity by working their factors more intensively, and finally perhaps achieve some super-normal level of employment. Finally, however, they must ration output in some way, through delivery delays, refusal to take orders, and so on, forcing people to accumulate more wealth than they want to.

Now we can look a little more closely at the introduction of unemployment into the two-sector production model. The simplest way to do this is to add a term  $u$ , the proportion of the labor force unemployed to the labor market clearing equation (1.4), so that it reads

$$(2.1) \quad l_c + l_I + u = 1.$$

If we do this the effect in equation (1.3) is to increase the capital-employed labor ratio by a factor of  $1/(1-u)$ . The capital intensities in the two sectors are still determined by (1.1) - (1.2). Unfortunately raising the capital-labor ratio in the two sector model where  $k_c > k_I$  has the effect of increasing the rate of output in the consumption goods sector. Since the original problem was an excess flow supply of consumption goods, unemployment in this case moves us even further from equilibrium.

This paradox suggests a more symmetrical and more plausible specification: that capital as well as labor should be disemployed. To model this we need the idea of the capital intensity of unemployment, that is the ratio of capital disemployed to labor disemployed. Calling this  $k_u$ , equation (1.3) becomes

$$(2.2) \quad l_c k_c + l_I k_I + u k_u = k.$$

It is hard to think of a good theory of the capital intensity of unemployment. The situation is much simplified if we assume that  $k_u$  is always equal to  $k_c$ , the capital intensity in the consumption sector. In this case, the effect of unemployment is to leave the rate of investment unchanged for any given  $p_k$ , and to reduce the flow rate of output of consumption goods. The economy acts as if a certain proportion of its consumption-producing capacity did not exist. The flow supply of investment remains  $q_I(k, p_k)$ , the flow supply of consumption becomes  $q_c(k, p_k, u)$  with  $\partial q_c / \partial u < 0$ .

Unemployment of this kind also has repercussions in the assets markets. First there is the classic effect that as real income declines the transactions demand for money also declines, and this by itself tends to lower the bond interest rate and the rate of return to capital.

But there is another effect, which is largely ignored in the literature, although it seems to me to be closely related to the accelerator. When capital is unemployed we must imagine that unemployment is distributed randomly for short periods over all existing capital and is not concentrated permanently like some curse on specific machines. The same thing, of course, is true for labor. This means that an owner of capital must expect his capital to be unemployed and earning no rent a certain proportion of the time depending on the unemployment rate and the capital intensity of unemployment. Rentals to

capital as an asset will now depend on both  $p_k$  and  $u$ , with  $\partial r/\partial u < 0$ .

Equations (1.5) - (1.7) must now read

$$(2.3) \quad \hat{p}_m m = L(a, q(k, p_k, u), \pi_m, i + \pi_m, \frac{r(p_k, u)}{p_k} + \pi_k)$$

$$(2.4) \quad \hat{p}_m b = H(a, q(k, p_k, u), \pi_m, i + \pi_m, \frac{r(p_k, u)}{p_k} + \pi_k)$$

$$(2.5) \quad \hat{p}_k k = J(a, q(k, p_k, u), \pi_m, i + \pi_m, \frac{r(p_k, u)}{p_k} + \pi_k)$$

This effect tends to lower the relative price of capital as unemployment rises, and might be called the static accelerator.

In the assets markets, then, unemployment has two offsetting effects. First, by lowering real income, it reduces the transactions demand for money, interest rates and the rate of return to capital; second, by lowering the expected profitability of capital it tends to lower the price of capital. Of these effects the second appears to be more important in modern economies. Recessions drastically affect corporate profits and in turn the prices people will pay to hold assets whose yield reflects profits, like common stock.

The consumption market clearing equation becomes

$$(2.6) \quad q_c(k, p_k, u) = c^d(a, q_c(k, p_k, u) + p_k q_I(k, p_k) + d \hat{p}_m - e \\ + \pi_m \hat{p}_m g + \pi_k p_k k) + e.$$

As  $u$  rises the  $p_k$  required to clear the consumption goods market falls, since the effect of rising  $u$  is to reduce excess supply. (A rise in  $u$  reduces  $q_c$  by the same amount on the left and right hand sides

of (2.6), but the marginal propensity to consume out of disposable income is less than one, so that the fall in supply is larger than the fall in demand.)

We can graph the aa and cc curves conveniently in  $(u, p_k)$  space. The cc is always downward sloping, but the aa may be upward or downward sloping depending on whether the effect of lower transactions demand for money or the rentals effect predominates in the assets markets.

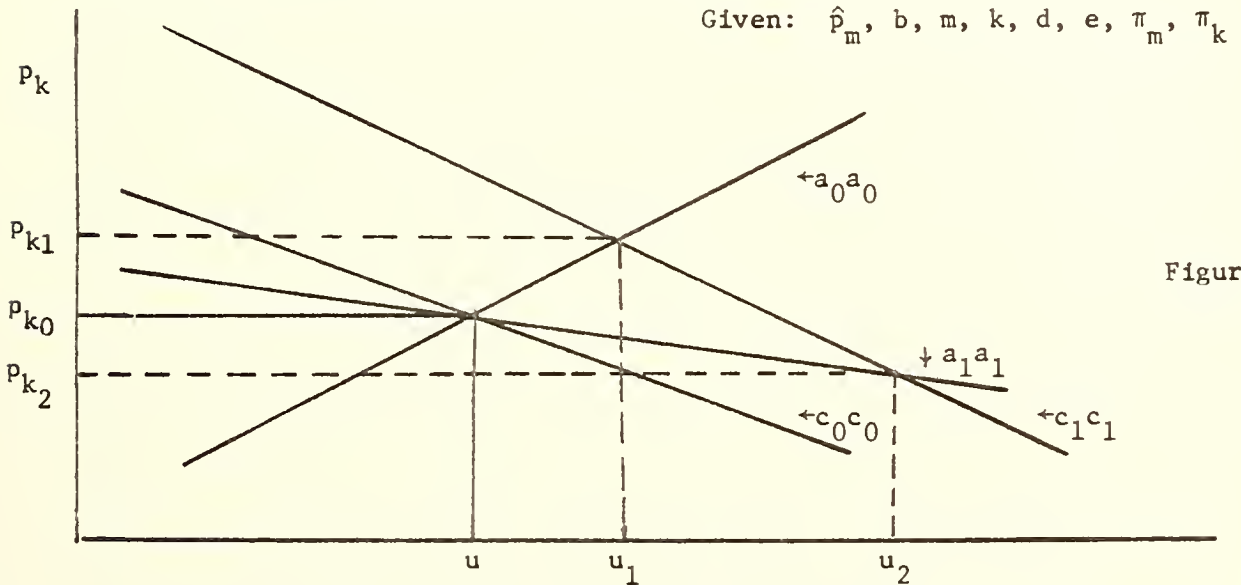


Figure 2-3

It is easy to see from Figure 2-3 that the slope of the aa curve (which it should now be clear plays the role of the LM) determines whether the multiplier consequences of a given deflationary change (say lower government expenditures) will be partially offset by higher investment (as with  $a_0 a_0$ ) or will be exacerbated by the collapse of profits expectations and investment (as with  $a_1 a_1$ ). The same ideas apply to the idea of "pump-priming" to restore business confidence and levels of investment. If the aa in  $(u, p_k)$  space is downward sloping, expansionary fiscal policy will

be enhanced by rising profit levels and a higher rate of investment.

The reader can verify that if the  $aa$  is flat, the effects of changes in government policies on real income follow the usual multiplier rules. Remember in doing this that a rise in  $e$  alone is a balanced budget increase in expenditures, and that a simultaneous increase in  $d$  and  $e$  corresponds to a deficit-financed increase in government expenditure.

This kind of instantaneous unemployment equilibrium does not necessarily involve a wrong level of real wages. Both rentals and wages to employed factors are exactly at their full-employment equilibrium level for the  $p_k$  finally arrived at. The trouble lies in money prices, or in the relative price of capital, that is, the incentive to investment. The value of money is too low, or the incentives to invest too weak to achieve full employment. It is no good for unemployed labor to try to bid down the real wage, since the problem is in money wages and money prices. In traditional Keynesian models, it seems to me, attention is arbitrarily and futilely focussed on the real wage of labor as an important parameter in unemployment. This real wage differs from the full employment real wage only because of the arbitrary specification of full employment of capital, an assumption which is neither theoretically important nor empirically justified. Whatever goes wrong with the market for labor can afflict the market for capital services as well. What is wrong is that the money price level for consumption goods has for some reason failed to be at the level compatible with full employment.

Section 3

In the last section I dealt only with the problem of momentary, or instantaneous equilibrium. This same model can be embedded in a differential equation system. This system begins (as did the full-employment system (1.10) - (1.14)) by requiring the assets markets to be cleared at each instant:

$$(3.1) \quad \hat{p}_m m = L(a, q(k, p_k, u), \pi_m, i + \pi_m, \frac{r(p_k, u)}{p_k} + \pi_k)$$

$$(3.2) \quad p_k k = J(a, q(k, p_k, u), \pi_m, i + \pi_m, \frac{r(p_k, u)}{p_k} + \pi_k)$$

The symbol  $\hat{p}_m$  now stands for the actual value of money. The consumption goods market becomes

$$(3.3) \quad q_c(k, p_k, u) = c^d(a, q(k, p_k, u) + d \hat{p}_m - e + \pi_m \hat{p}_m g + \pi_k p_k k) + e.$$

These three equations determine  $p_k$ ,  $i$ , and  $u$  given  $\hat{p}_m$ ,  $m$ ,  $b$ ,  $d$ ,  $e$ ,  $k$ ,  $\pi_m$  and  $\pi_k$ . Because I have assumed that the capital intensity of unemployment is the same as the capital intensity in the consumption goods sector, the law of accumulation of capital remains unchanged and does not involve the unemployment rate. The law of accumulation of the government debt is not affected by unemployment.

$$(3.4) \quad \dot{k} = q_I(k, p_k) - n k.$$

$$(3.5) \quad \dot{g} = d - n g.$$

Again, these five equations must be supplemented by two equations determining expectations ( $\pi_m$  and  $\pi_k$ ), and three specifying paths or goals for the three government policy tools,  $d$ ,  $e$ , and the ratio of  $m$  to  $b$ . But now I need one more equation to close the system. This equation must be a theory of the actual value of money. For example, the full employment model

can be written by requiring  $u$  to stay fixed at some  $u^*$ ; perhaps zero.

$$(3.6) \quad u = u^*.$$

The Phillips' Curve theory in its simplest form relates the rate of change in  $\hat{p}_m$  to  $u$ .

$$(3.7) \quad \frac{\dot{\hat{p}}_m}{\hat{p}_m} = f(u).$$

At this point we must consider what I will loosely call the "reality" that lies behind this kind of model. In particular, why doesn't  $\hat{p}_m$  always adjust to  $p_m^*$ , the full-employment value of money? (I am here assuming at least a rough and accepted division between normal or frictional unemployment and excessive or involuntary unemployment.)

In reality, both the  $aa$  and  $cc$  schedules have large random components. The  $cc$  moves around because of uneven technical progress, random changes in tastes, strikes, bad harvests, and so on; the  $aa$  moves with expectations, alterations of liquidity preference and changing confidence in capitalist institutions. Both, as well, are subject to large random movements of the government policy tools. Some of these policy changes are designed to offset the other movements, but some, arising from wars, politics, bad forecasts, mistaken theories and other reasons of state, add a substantial random component to the  $aa$  and  $cc$  curves, and hence to the full-employment value of money.

If it were possible to observe the full-employment value of money as an economic time series I would therefore expect it to exhibit frequent large changes which were frequently reversed. The actual value of money, on the other hand, is not a series I expect to be capable of frequent large



movements. It is the result of thousands or tens of thousands of decisions, each depending on a slow diffusion of information about market conditions. The precise parameters of this slow diffusion could be specified only in the context of a model of exchange that itself took account of the stochastic structure of actual trading. Even without such a model it does not seem to me surprising that the actual value of money fails to follow each wriggle of the full employment value of money faithfully.

In fact, looking at the situation in these terms suggests that a fruitful hypothesis may be that the actual value of money behaves as if it were trying to predict the current value of the full-employment value of money. This hypothesis is a rich one; I will try to show in the next section that it includes many currently proposed theories of the price level as special cases. It also creates a systematic program for developing theories of the actual price level which may go beyond currently proposed models.

I propose, then, that we should look on the full-employment value of money as a stochastic process which the actual value of money is trying to predict. As has been proposed before (see Muth [5] and Whittle [6]), we may imagine that the economy predicts rationally, that is, that the process that converts the full-employment value of money series into the actual value of money series is based on the actual statistical characteristics of the former series, and tries in some sense to minimize its errors given its information.

It is often said that Keynesian theories of unemployment depend on price or wage rigidity to produce unemployment. I think we must be a little careful in our thinking on this point once we have a dynamic macroeconomic model.

Unemployment in this model is due to a failure of the actual value of money to equal the full-employment value of money, but this "rigidity" really amounts to a failure of the actual value of money to jump instantaneously to the full-employment value. The actual price level may be flexible over time; may, in fact, be adjusting toward the full-employment value, and still fail in a given instant to equal it. We can distinguish two senses of price flexibility. The first is instantaneous movement of money prices to the full-employment level. This is a very strong condition and I think few people would expect it to hold in any real economy. The second is a systematic tendency of actual prices to move toward their full-employment level. In this version of price flexibility everything turns on the exact dynamic adjustment process, whether the average lag is measured in months or years and so forth. There does not seem to me to be any good argument that the parameters of this adjustment process should depend on market organization variables like monopoly or unionization. Monopoly in factor and output markets will have important and well-understood consequences for relative prices and the distribution of wealth, but I have seen no cogent argument linking monopoly to money price dynamics.

Section 4

Suppose, then, that we have the full-employment value of money  $\{p_m^*(t)\}$ , a stochastic process, and that we assume that the actual value of money  $\hat{p}_m(t)$  is a functional of past full-employment values. The unemployment rate is a function of  $p_m^*$ ,  $\hat{p}_m$ ,  $k$ ,  $g$ ,  $\pi_m$ ,  $\pi_k$ ,  $d$ ,  $e$  and  $m$ . (In fact, as I pointed out in the last section the unemployment rate and productivity will themselves evolve through some dynamic adjustment process. I ignore that complication in this paper, and assume that  $u$  adjusts instantaneously to clear the market for consumption goods, while productivity of employed factors stays constant.)

As a heuristic hypothesis I assume that  $\hat{p}_m(t)$  is a rational predictor of  $p_m^*(t)$ , that it tries to minimize the expected squared deviation of  $\hat{p}_m$  from  $p_m^*$ . The theory of such rational predictors is fairly well-developed, but unfortunately I have not mastered it completely. What follows is a sketchy discussion of some thoughts arising from this hypothesis.

The best predictor of a stochastic process depends on the statistical characteristics of the process. The theory is best developed for stationary processes, that is, where the expected value of any observation of the process is a constant independent of time and the covariance between any two observations depends only on the difference in the dates of observation, and is independent of the absolute date. For at least a certain class of stationary processes (those that can be represented as a linear weighted average of another process which is a series of uncorrelated normal random variables,

often called white noise) the best predictors are themselves linear weighted averages of past observations. If the process  $\{p_m^*(t)\}$  had this structure, we might expect that  $\hat{p}_m(t)$  could be described as

$$(4.1) \quad \hat{p}_m(t) = \int_{-\infty}^t w(t - \tau) p_m^*(\tau) d\tau$$

where  $w(\tau)$  is a positive function defined for  $0 < \tau < \infty$  which depends on the exact statistical properties of  $\{p_m^*(t)\}$ . If  $\{p_m^*(t)\}$  were a first-order auto regressive process then  $w(\tau)$  would have the familiar adaptive expectations form:

$$(4.2) \quad w(\tau) = \beta e^{-\beta\tau} \quad 0 < \tau < \infty.$$

This system allows for unemployment (and excess demand) because the actual value of money lags behind movements in the full-employment price of money. It would also exhibit Phillips' Curve behavior, since the rate of change of  $\hat{p}_m$  is proportional to the gap between  $\hat{p}_m$  and  $p_m^*$ , and the unemployment rate will depend, given other variables, on the same gap.

This adaptive expectations mechanism has the additional defect that it cannot allow for simultaneous unemployment and inflation. This is because whenever  $p_m^*$  is higher than  $\hat{p}_m$ ,  $\hat{p}_m$  itself will be rising. The differential equation version of (4.2) is

$$(4.3) \quad \dot{\hat{p}}_m / \hat{p}_m = \beta(p_m^* - \hat{p}_m).$$

However other contours for  $w(\tau)$  could reconcile inflation and unemployment. If  $w(\tau)$  has the shape indicated in Figure 4-1

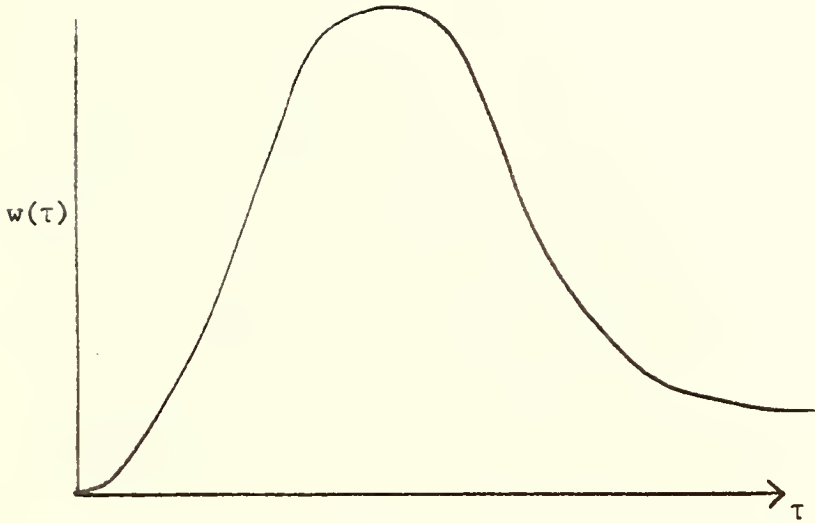


Figure 4-1

it would be possible to have both inflation and unemployment. If, for example, a war forced a sudden sharp fall in  $p_m^*$ , the full employment value of money (an increase in the full-employment price level) the initial effect would be a gradual fall in  $\hat{p}_m$ . If after some time the government tried to fight the inflation by raising  $p_m^*$  it could drive  $p_m^*$  above  $\hat{p}_m$ , thereby causing unemployment, but  $\hat{p}_m$  would continue to fall for some time because of the heavy weight  $w(\tau)$  puts on past values of  $p_m^*$ . Once they have achieved some unemployment the authorities come under political pressure to limit the severity of the recession. Thus it is possible to imagine a time path like Figure 4-2, where  $\hat{p}_m$  itself never actually rises.

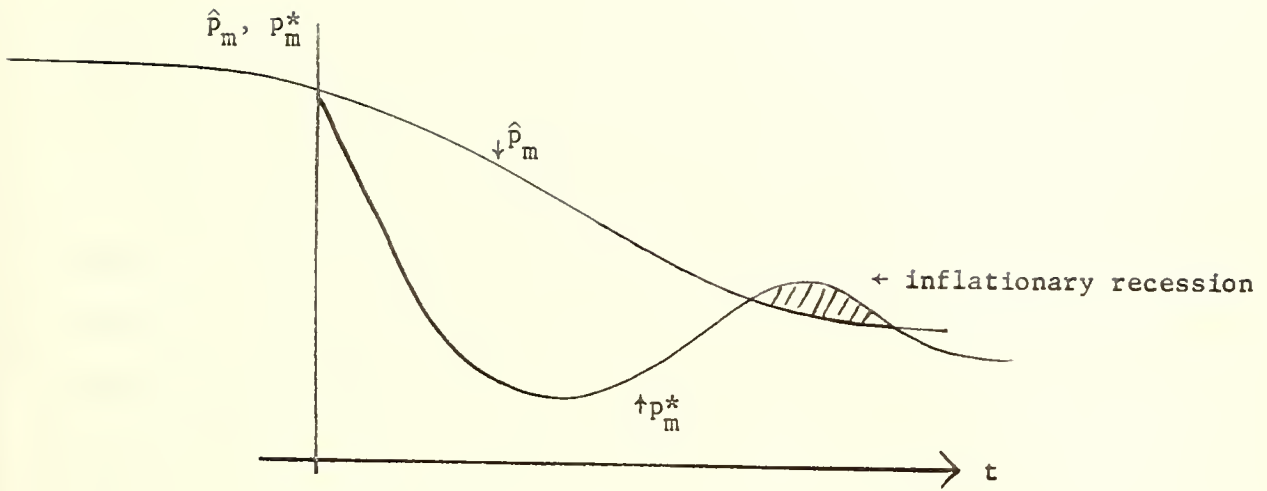


Figure 4-2

All linear schemes of the form (4.1) have the feature that they exhibit a long-run nonvertical Phillips Curve. In other words, a change in the steady-state rate of inflation will also alter the steady-state rate of unemployment. This is because if the steady-state rate of inflation is different from zero,  $\hat{p}_m$  will never catch up with  $p_m^*$ , and the size of the permanent gap between them will depend on the steady-state rate of inflation. (Notice that the rate of change of  $\hat{p}_m$  will converge to the average rate of change of  $p_m^*$ . The gap is between their levels.) Many people find this to be an unacceptable feature of a theory, on the grounds that economic agents cannot be fooled forever, so that  $\hat{p}_m$  will, if the rate of increase of  $p_m^*$  is maintained long enough, converge to  $p_m^*$ , not trail mechanically behind it.

The difficulty is, of course, that at least in economies with fiduciary money and political control over monetary and credit policy, the full-

employment value of money is not likely to be a stationary stochastic process. The hypothesis of stationarity may have been more plausible as a stylization of reality under the gold standard before 1914, but is now untenable.

This leads to a final question, which I can pose but cannot answer at present. What is a reasonable statistical specification for the full-employment value of money, and what is the rational predictor for such a process? This approach at least outlines a way of generating hypotheses about price-level dynamics. Each statistical specification for the underlying stochastic process  $\{p_m^*(t)\}$  systematically generates a rational predictor which is at the same time a theory of the dynamics of the actual value of money.

### Conclusion

In this paper I have tried to describe a way of looking at macroeconomic problems and their interaction. In particular I have attempted to get straight in the context of one particular model the relation between unemployment, price and wage flexibility, and money price dynamics. Although the individual elements of the argument are straightforward (and, as some readers may think, too well-known to require repeating) the connection of them in this way leads to some particular conclusions and to a program for econometric research.

This analysis goes against the usual theories of the Phillips Curve, and especially against the notion of cost push inflation. In that sense it supports the classical or monetarist side of recent controversies. It does not however endorse the idea of speedy automatic adjustment to full employment with-

out government policy intervention. In fact the role of government intervention appears in this light to be many sided: both cause-of and cure for macro-economic maladies. I hope that I have proposed a sufficiently concrete program of research into price dynamics to satisfy Keynesians who dislike the mysticism of some monetarist writing.

The great difficulty is in observing the full-employment value of money. This is, of course, a theoretical construct, and stands or falls by its helpfulness in describing and predicting reality. Something like it could, I assume, be generated using existing large econometric models, if they have sufficiently well-articulated financial sectors, though elaborate lag structures tend to confuse the issue.

Duncan K. Foley

July 18, 1971



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